Effect of Masks on Indoor Classroom Covid-19 Transmission

Hongjia Huang (hh3043@nyu.edu)*

Abstract

This paper mainly analyze the effect of face mask on slowing down the Covid-19 transmission in indoor classrooms from the perspective of masks' thickness and discuss how different material of mask may have different impact on this effect. The main finding is that thicker face masks could slow down Covid-19 transmission further, with different material having different slowing/resisting power against the virus spread.

1 Introduction

Bazant and Bush (2021) investigates how different factors affect the likelihood of Covid-19 transmission in an indoor case. However, it does not focus much on the effect of wearing masks. In this paper, the effect of mask is generally introduced as penetration rate which is only mentioned briefly and given as experiment results. Liu et al. (2021) dives deeper into the characteristics of filter media/material in general and in particular gives a calculation formula for the penetration rate. With some simplication on the structure/component of mask, we could use the same formula for masks as well. Putting these two papers together, we could glimpse more on the effect of masks in protecting people against virus transmission.

This report aims to address the question:

> *How does the role of masks affect the spread of Covid-19 in an indoor classroom?*

I plan to investigate this as follows: first, derive a formula between spread and characteristics of masks (here we only consider material and thickness). Then, takes certain material as example and see the results from my derived formula on the effect of thickness as well as brefly discusssing how different material may differ this impact.

2 Problem setup

According to Bazant and Bush (2021), with time τ being quite long comparing to the time required for air to change/refresh, we can derive a formula for the equilibrium state:

$$\frac{\bar{\beta}_a}{s_r} = p_m^2 f_d \lambda_q, \tag{1}$$

where $\bar{\beta}_a$ is the mean number of transmissions per time per infectious individual (people infected) per susceptible individual (people that could be infected) with s_r as a rescaling factor. p_m is the penetration rate of mask, with f_d as the ratio of the concentration of infection quanta ("infectiousness") in the well-mixed room to that in the unfiltered breath of an infected person. λ_q is expressed as the rate of quanta of exhaled air by an individual. $\alpha = \frac{\bar{\beta}_a}{s_r}$ is an important measurement for the severity of Covid spread by reflecting how infective the classroom is. However,

*Partner: Zhaodong Liu (zl4789@nyu.edu)

it is indirect and a more direct measurement may be needed.

we could also derive a bound for the total time τ required to reach the tolerance rate ϵ for people to become infected given with N people in the classroom. Ignoring air filtration (air change created by air filters) and pathogen deactivation, from Bazant and Bush (2021) we could get:

$$N\tau < \epsilon \frac{\lambda_a V}{Q_b^2 p_m^2 C_q s_r} \tag{2}$$

Where Q_b is the breathing flow rate (exhaled volume per time), C_q is the concentration of exhaled infection quanta by an infectious individual, λ_a is the outdoor air exchange rate, V is the volumn for the classroom,

We could rewrite this equation as the following:

$$\tau < \epsilon \frac{\lambda_a V}{Q_b^2 p_m^2 C_q s_r N} = \tau_m \tag{3}$$

Where we acquire an upper bound for τ, denoted as τ_m.τ_m could be viewed as the longest time for the Covid spread to be considered "in control" with the preset risk tolerance rate ϵ. This is very straight forward in measuring Covid spread.

Then, we consider the function given by Liu et al. (2021) about the penetration rate of particles through some media that serves as filter:

$$P_m = \exp\left(-\frac{4\rho\eta L}{\pi D_f(1-\rho)}\right) \tag{4}$$

Here L denotes thickness of the media, the D_f, ρ and η are all determined by the given media that we are using to filter the particles. We could directly apply this formula to masks given the simplification that we view a mask as a thin layer that is made up of one material (media) and filters out droplet which contains virus.

With these given formula, I derived some analytic results and I would show them in the section below along with a plot for visualization on the result for different mask thickness. In particular, I would use α and τ_m as 2 evaluation metric for the effectiveness of mask in indoor Covid spread.

3 Results

We could rewrite equation (4) as:

$$p_m = e^{-\gamma L} \tag{5}$$

with $\gamma = \frac{4\rho\eta}{\pi D_f(1-\rho)}$ a constant that is entirely related to the given mask material.

Plugging equation(5) into (1) and (3), we can get:

$$\alpha = \frac{\bar{\beta}_a}{s_r} = e^{-2\gamma L} f_d \lambda_q \tag{6}$$

$$\tau < \epsilon \frac{\lambda_a V}{Q_b^2 e^{-2\gamma L} C_q s_r N} = \tau_m \tag{7}$$

This means that

$$\alpha \propto e^{-2\gamma L} \quad \text{and} \quad \tau_m \propto e^{2\gamma L}$$

This means that with the same material given, thicker mask gives lower α, and larger τ_m, which corresponds to lower infectiveness and longer time to reach the tolerated risk ϵ, thus demonstrating better performance in slowing down transmission. With penetration rate and thickness given, we can further compute γ via equation (5). As we can see from this equation, given a fixed thickness, stronger material should have lower penetration rate, thus correspond to larger γ.

Furthermore, I use a similar plot to that of Fig.3 in Bazant and Bush (2021) to visualize how thickness effect the $N - \tau_m$ relationship via level sets. For the below plot, we assume $\gamma = 1$ for simplification, use the same given parameters as the classroom case:

$\epsilon = 10\%$
$\lambda_a = 8.0\,\mathrm{h}^{-1}$
$V = 301\,\mathrm{m}^3$
$Q_b = 0.5\,\mathrm{m}^3\,\mathrm{h}^{-1}$
$C_q = 30\,\mathrm{quanta}\,\mathrm{m}^{-3}$
$s_r = 25\%$

and with the function

$$\epsilon \frac{\lambda_a V}{Q_b^2 e^{-2L} C_q s_r} = N\tau_m \tag{8}$$

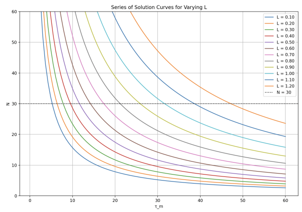

Figure 1: Level sets for N-τ_m Space

As L increase linearly, the curve moves up into the top-right corner faster and faster. In particular, consider fixing N to 30, the corresponding two τ_m value for L = 0.2 and L = 1.2 is 6.386 and 47.188. The increase rate per unit thickness for τ_m is $e^{1.85}$ ($\approx e^2$). if we consider other values for γ, then the rate of increase for τ_m per unit thickness would become $e^{2\gamma}$ for the more general case.

4 Summary

So far, I have derived formulas that involves the thickness of mask as well as material characteristics of masks (expressed as γ) in the measurement of indoor Covid-19 transmission. The result is that both thickness L and mask material characteristic γ have a exponential relationship with evaluation metric α and τ_m (see equation (6) and (7)). Thicker mask correspond to better performance in slowing down the transmission. Also, better material correspond to a larger γ value. γ may in turns have the potential for measuring the effectiveness of different mask material in preventing infection.

The model I derived shows the importance of mask wearing in Covid spread and also that different mask has different effect. However, in this work, I only considered the simplest case that everyone wear the same kind of mask, this may not be always true in real world examples. Also, the simplification of mask as a layer of the same material ignores more complicated structures in mask design and may potentially lead to a very rough estimation to the real world examples.

References

Bazant, M. Z. and Bush, J. W. M. (2021). A guideline to limit indoor airborne transmission of COVID-19. *Proceedings of the National Academy of Sciences*, 118:e2018995118.

Liu, Z., Chen, D.-R., Wang, P., and Ji, Z. (2021). Effect of filtration pressure on the particle penetration efficiency of fibrous filter media. *Separation and Purification Technology*, 274:119086.